The Rambam Code

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In this note we analyze the famous Maimonides so-called Torah Code.

Maimonides is Rabbi Moses Ben Maimon who is also known as רמבם, the Rambam, for short. He lived in Egypt in the twelfth century, 1135-1204. He was a philosopher, a physician, a halakhist, and a medical writer. He held the position of being the physician in the court of Al-Fadhil, the vizier of Egypt under Saladin. And as well, he was the head of the Jewish religous community in Cairo. Among his religious writings is the famous משנה תורה, *Mishneh Torah*, an organized compendium of the entire halakhah, the laws associated with the 613 commandments followed by observant Jews.

In the section of Exodus discussing the observance of the Passover the following Torah code for the two key words משנה תורה can be found. Each code instance has a skip interval of 50 and from the מ of משנה to the letter preceeding the ת of Torah is exactly 613 letters. This is illustrated in the code array of figure 1.

Having observed this phenomena, we could ask what is the probability of this occuring. To answer this question we have to put the question in the context of a protocol in which we must first define a population of texts and then describe the search experiment that is to be performed in the population of texts.

For this example code, our text population consists of texts which are all possible positional permutations of the letters in the Torah text. The Torah text has 304,805 letters and the number of possible positional permutations is 304,805!, a very large number. One such permutation is the identity permutation so the Torah text is one of the texts in the population. Our experiment is that of drawing out at random one of the texts in the population and from a pre-specified list of text positions determining whether or not the letters משנהתורה occur in this order at the positions specified in the list Notice that the description of the experiment has two dimensions: that of selecting at random one of the texts in the population and the second the determination of whether in a pre-given list of text positions the letters משנהתורה occur in the order given.

For this text population and protocol of randomly selecting a text, we can easily compute the probability of observing the 8 letters משנהתורה in a given list of character positions. For this text population, the conditional probability of any letter in any character position given any other combination of letters in any other character position is equal to the marginal probability of the letter. So no matter what our list of letters may be and no matter what positions we specify the letters to occur in, the probability of observing the joint event is just

93861	אמריהוהאלמשהלאישמעאליכמפר	**93837**
93886	עהלמענרבותמופתיבארצמצרימו	**93862**
93911	משהואהרנעשואתכלהמפתימהאלה	**93887**
93936	לפניפרעהויחזקיהוהאתלבפרעה	**93912**
93961	ולאשלחאתבניישראלמארצוויאמ	**93937**
93986	ריהוהאלמשהואלאהרנבארצמצרי	**93962**
94011	מלאמרהחדשהזהלכמראשחדשימרא	**93987**
94036	שונהואלכמלחדשיהשנהדברואלכ	**94012**
94061	לעדתישראללאמרבעשרלחדשהזהו	**94037**
94086	יקחולהמאיששהלביתאבתשהלבית	**94062**
94111	ואמימעטהביתמהיותמשהולקחהו	**94087**
94136	אושכנוהקרבאלביתובמכסתנפשת	**94112**
94161	אישלפיאכלותכסועלהשהשהתמימ	**94137**
94186	זכרבנשנהיהילכממנהכבשימומ	**94162**
94211	נהעזימתקחווהיהלכמלמשמרתעד	**94187**
94236	ארבעהע ריומלחד הזהושחטואת	**94212**
94261	וכלקהלעדתישראלבינהערבימול	**94237**
94286	קחומנהדמונתנועלשתיהמזוזתו	**94262**
94311	עלהמשקופעלהבתימאשריאכלואת	**94287**
94336	ובהמואכלואתהבשרבלילההזהצל	**94312**
94361	יאשומצותעלמררימיאכלהואלתא	**94337**
94386	כלוממנונאובשלמבשלבמימכיאמ	**94362**
94411	צליאשראשועלכרעיוועלקרבוול	**94387**
94436	אתותירוממנועדבקרוהנתרממנו	**94412**
94461	עדבקרבאשתשרפווככהתאכלואתו	**94437**
94486	מתניכמחגרימנעליכמברגליכמו	**94462**
94511	מקלכמבידכמואכלתמאתובחפזונ	**94487**
94536	פסחהואליהוהועברתיבארצמצרי	**94512**
94561	מבלילההזהוהכיתיכלבכורבארצ	**94537**
94586	מצרימאדמועדבהמהובכלאלהימ	**94562**
94611	צרימאעשהשפטימאנייהוהוהיהה	**94587**

Fig. 1. Code array showing the close spatial relationship between the key words **רמבם**, Rambam, the short nick name by which Maimonides is known, and the title of his most famous book, *Mishneh Torah* **משנה תורה**. The numbers on the left and the right give the text character positions for the letters in the leftmost and rightmost columns of the code array.

the product of the marginal letter probabilities. Or simply stated, the event of observing any one letter in any character position is independent of observing any other letter in any other character position.

If we change the text population, we change the probability of the joint event. For example, consider the text population consisting of all permutations of the Torah text in which each permuted text has the property that each letter is part of a run of letters of its kind of length 50, with the exception of possibly each letter's last run. If we order the list of given character positions and in our ordered list each successive character position is more than 50 from the previous character position, then we have independence as before. But if the some of the successive character positions are less than 50, then independence does not hold and since the list of 8 letters משנהתורה does not have any successive repeating letters, the probability of observing them in their pre-specified positions in this new population will be zero.

Letter	Probability	Letter	Probability
א	0.088774	ל	0.070766
ב	0.053624	מ	0.082314
ג	0.006919	נ	0.046351
ד	0.023070	ס	0.006014
ה	0.092045	ע	0.036909
ו	0.100106	פ	0.015764
ז	0.007211	צ	0.012998
ח	0.023585	ק	0.015403
ט	0.005918	ר	0.059464
י	0.103446	ש	0.051164
כ	0.039264	ת	0.058890

Table 1. Lists the letters and their probabilities as they occur in Torah.

Table 1 gives the letter frequency of each letter for the five books of the Torah. The probability p of observing the 8 letters משנהתורה in any given fixed set of text positions is computed as

$$\begin{aligned} p &= 0.082314 \times 0.051164 \times 0.046351 \times 0.092045 \times \\ &\quad 0.058890 \times 0.100106 \times 0.059464 \times 0.092045 \\ &= 5.79767 \times 10^{-10} \end{aligned}$$

a very small probability indeed.

Because it is so small we might think that this is an unusual event. But the way in which we might naively think that this is an unusual event could be very wrong. To understand this, we have to understand the meaning of the probability p we computed. It means this: If we were to sample one text from a population of all texts which have the same number of letters of each kind that the Torah has and if we were to designate a list of 8 particular text positions, the probability is p that we would discover the letters משנהתורה in the designated places, precisely in this order, That is, in such an experiment, the probability is very small that we would find משנהתורה.

The problem is that the given positions specified in our list, did not come about a priori. We saw the example code first. And now we ask what is the probability of observing precisely this pattern. The difficulty is that after we saw the precise pattern, it is meaningless to ask the question about what would be the probability of observing that exact pattern in the Torah text.

To get to a meaningful question, we have to form a broad class of patterns of which the pattern we observe is a member. We can ask the question about the probability of sampling a text from the text population and observing that the text has in it belonging to the specified pattern class. If the pattern class is sufficiently broad, we may use it after the fact to inquire about a demonstrably a priori experiment.

Suppose we select a sampling pattern of exactly a skip interval of 50 for משנה and 50 for תורה with 613 letters in between the מ of משנה and the ת of תורה. This specifies for what we are going to look: a particular pattern of 8 letters having a span of some 1+613+1+150 = 765 character positions. Now we perform the experiment. We take the Torah text having some 304,805 letters, and look in all the 304,805 - 765 +1 = 304,041 placements in which the pattern span of 765 character positions can be placed and look to find an occurrence of the pattern משנה תורה. What is the probability of observing no occurrences? What is the probability of observing one occurrence? two occurrences? three occurrence? and so on.

The probability of observing the pattern משנה תורה in any one placement is 5.79767×10^{-10}. There are 304,041 possible placements in which to observe a pattern that spans 765 characters. So the expected number of times or mean number of times m that we would observe the code משנה תורה in such a text is

$$m = 5.79767 \times 10^{-10} \times 304,041 = 1.76273 \times 10^{-4}$$

Assuming that the number of times that we observe the pattern is Poisson distributed, the probability q of observing the pattern k times is

$$q = \frac{\exp(-m)m^k}{k!} \tag{1}$$

To determine the probability of not observing the pattern at all we take $k = 0$ in equation 1. When $k = 0$, this probability is $\exp(-m)$, which for m near 0 is approximately $1-m$. The probability of not observing the pattern 0 times is the probability of observing it at least once and this is $1-(1-m) = m$. Hence for this experiment, the probability of observing משנהתורה at least once in code is 1.76273×10^{-4}.

The probability of observing the character sequence רמבם in any given placement is

$$0.059464 \times 0.082314 \times 0.053624 \times 0.082314 = 2.16056 \times 10^{-5}$$

The number M of possible placements in a text of length N characters, searching over skip intervals from D_{min} to D_{max} is

$$\begin{aligned} M &= \sum_{d=D_{min}}^{D_{max}} [N - (L-1)d] \qquad (2) \\ &= (D_{max} - D_{min} + 1)[N - \frac{(L-1)(D_{max} + D_{min})}{2}] \end{aligned}$$

Taking $N = 765$, $D_{min} = 2$, and $D_{max} = 254$, we obtain $M = 192,786$. The expected number m of occurrences is then

$$m = 192,768 \times 2.15056 \times 10^{-5} = 4.14559$$

And the probability of observing at least one occurrence is then $1 - \exp(-m) = 0.98416$. Hence the probability of observing the letter sequence משנה at a skip interval of 50, the letter sequence of תורה at a skip interval of 50, and 613 letters between the מ of משנה and the ת of תורה, and the letter sequence רמבם, searching over skip intervals of 2 to 254, within the 765 character span of the pattern משנהתורה is $1.76273 \times 10^{-4} \times .98416 = 1.734808 \times 10^{-4}$

If we have not peeked at the text first to see if it has the pattern we are looking for, then the Torah text itself can be considered as a randomly selected or arbitrarily selected text. And the probability of 1.734808×10^{-4} applies to it. However, it we snooped first looking for some pattern, and we discovered something we think is interesting, and then we do the probability calculation, the computed probability only means: If we were to select a text at random from the population and if we were to look at this randomly selected text in all the possible placements of the pattern with the skip of 50 and the interval of 613, checking each placement to see if it has in the given order the eight letters of the two key words, then the probability is 1.734808×10^{-4} that we will find in that text at least one pattern instance. But in general this probability does not apply to the Torah text because snooping at it first disqualifies it for being a randomly selected text, unless we can argue in that the pattern we based the probability calculation on is demonstrably a priori and does not contain in it any irrelevant detail.

The question is whether the class we specified is indeed demonstrably a priori. Is there anything in the class we specified that is not in an a priori manner relevant to the class specification? For example the name of the book and the fact that the book is a discussion of the 613 mitzvot is essential to the class. But is the skip of 50 essential? Would we find it just as relevant to find the pattern with a skip of 51 or 117 or 2007? If we do not have an a priori reason for keeping the skip of 50 in the class specification, then we do not have a demonstrably a priori specification.

Now suppose that for our pattern משנהתורה we do not specify a skip interval of 50. Suppose we specify any skip interval of say 1 to D. Then everything changes. For a skip interval of d, the span of the pattern span is $615 + 3d$ character positions. The number of places such a pattern can be placed in a text of length 304,805 characters is $304,805 - (615 + 3d) + 1 = 304,190 - 3d$. Taking the sum for d being between 1 and D the total number M of possible pattern placements is

$$\begin{aligned} M &= \sum_{d=1}^{D} (304,190 - 3d) \\ &= 304,190D - 3(D)(D+1)/2 \end{aligned}$$

When $D = 1000$, there results

$$\begin{aligned} M &= 304,190,000 - 3(1000)(1001)/2 \\ &= 302,688,500 \end{aligned}$$

Hence the expected number of times m that we observe the pattern משנה תורה is

$$m = 5.79767 \times 10^{-10} \times 302,688,500 = 0.175489$$

Now the probability that we observe the pattern at least once is $1 - \exp(-m) = 0.16095$, a chance of approximately one out of six times. This would certainly not be a rare event.

Remember what this probability means. If we take a text at random from the population and if we look at that text in all the possible placements of the pattern, checking each placement to see if it has in the given order and relative placement the eight letters of the two key words, then the probability is 0.16095 that we will find in that text at least one instance.

So if we have in hand a text with this pattern, we should not be surprised. Nothing unusual has occurred.